



GOVERNMENT OF KARNATAKA
KARNATAKA SCHOOL EXAMINATION AND ASSESSMENT BOARD

II YEAR PUC EXAMINATION MARCH 2023

SCHEME OF VALUATION

Subject Code: **35**

Subject: **Mathematics**

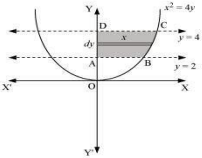
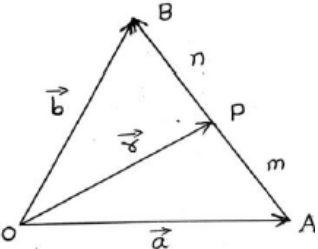
Instructions:

- a) Any answer by alternate method should be valued and suitably awarded.
b) All answers (including extra, stuck off and repeated) should be valued. Answers with maximum marks must be considered.

Qn No	PART A	Marks
1	b) or Writing Symmetric	1
2	c) or Writing $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	1
3	b) or Writing 6	1
4	a) Or Writing $ A ^{n-1}$	1
5	b) or Writing (-1, 1)	1
6	d) or Writing $e^x \sec x + c$	1
7	a) or Writing $\left(\frac{i+j+2k}{\sqrt{6}}\right)$	1
8	d) or Writing $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$	1
9	d) or Writing 120	1
10	c) or Writing P(S)	1
II		
11	2	1
12	11	1
13	0	1
14	4	1
15	3	1
III		
16	$5 \star 7 = 35$	1
17	Writing, $\frac{dy}{dx} = \cos(x^2+5)2x$ OR Writing $f'(x) = \cos(x^2 + 5)2x$	1
18	Two or more vectors having the same initial point	1
19	The common region determined by all the constraints including non-negative constraints of a LPP is called feasible region	1
20	$\frac{3}{8}$	1

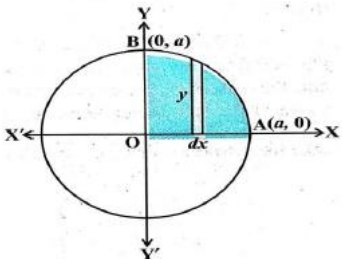
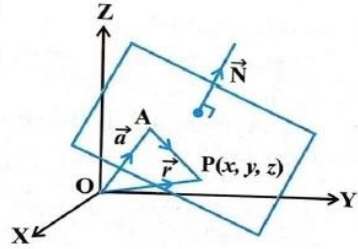
PART B		
21	Getting $(g \circ f)(x) = g(f(x)) = g(\cos x) = 3(\cos x)^2 = 3\cos^2 x$	1
	Getting $(f \circ g)(x) = f(g(x)) = f(3x^2) = \cos(3x^2)$	1
22	Writing $\sin^{-1} x = \theta$ and $x = \sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$	1
	Getting $\cos^{-1} x + \theta = \frac{\pi}{2}$ or $\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$	1
23	Writing $\sin\left(\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right)$	1
	Getting $\sin\frac{\pi}{2} = 1$	1
24	Writing $\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ OR $\text{Area} = \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$	1
	Getting $\text{Area} = \frac{15}{2}$	1
25	Getting $\frac{dy}{dx} + \cos y \frac{dy}{dx} = -\sin x$	1
	Getting $\frac{dy}{dx} = \frac{-\sin x}{1 + \cos y}$	1
26	Writing $y = x^x$ and $\log y = x \log x$	1
	Getting $\frac{dy}{dx} = y(1 + \log x)$ OR $\frac{dy}{dx} = x^x(1 + \log x)$	1
27	Writing $f(x) = \sqrt{x}$ and $f'(x) = \frac{1}{2\sqrt{x}}$	1
	Getting $\sqrt{25.3} = 5.03$	1
28	Writing $\int (2 \sec^2 x - 3 \sec x \tan x) dx$	1
	Getting $2 \tan x - 3 \sec x + c$	1
29	Getting $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx = [\tan^{-1} x]_1^{\sqrt{3}}$	1
	Getting $\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$	1
30	Writing $y^2 = 4ax$ and $2y \frac{dy}{dx} = 4a$	1
	Getting $2y \frac{dy}{dx} = \frac{y^2}{x}$ OR $y^2 - 2xy \frac{dy}{dx} = 0$	1
31	Getting $\vec{a} \cdot \vec{b} = 10$ and $ \vec{b} = \sqrt{6}$ OR	1
	Writing Projection of \vec{a} on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ \vec{b} }$	1
	Getting Projection of \vec{a} on \vec{b} is $\frac{10}{\sqrt{6}}$	1

32	Getting $\vec{a} \times \vec{b} = 20\hat{i} + 5\hat{j} - 5\hat{k}$	1									
	Getting Area of parallelogram = $\sqrt{450}$ sq units OR $15\sqrt{2}$ sq units	1									
33	Getting $\vec{b}_1 \cdot \vec{b}_2 = 19$ and $ \vec{b}_1 = 7, \vec{b}_2 = 3$ OR Writing $\cos\theta = \frac{ \vec{b}_1 \cdot \vec{b}_2 }{ \vec{b}_1 \vec{b}_2 }$	1									
	Getting $\theta = \cos^{-1}\left(\frac{19}{21}\right)$	1									
34	Writing $S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$	1									
	Getting <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>P(X)</td> <td>$\frac{1}{8}$</td> <td>$\frac{3}{8}$</td> <td>$\frac{3}{8}$</td> <td>$\frac{1}{8}$</td> </tr> </table> OR Writing the table allot 2 marks	X	0	1	2	3	P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
X	0	1	2	3							
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$							
PART-C											
35	Reflexive: Every triangle is congruent to itself	1									
	Showing R is symmetric	1									
	Showing R is transitive and hence R is an equivalence relation	1									
36	Getting $2 \tan^{-1} \frac{1}{2} = \tan^{-1} \frac{4}{3}$	1									
	Writing $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ OR $\tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{7} = \tan^{-1} \left(\frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \times \frac{1}{7}} \right)$	1									
	Getting $\tan^{-1} \frac{31}{17}$	1									
37	Writing $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$	1									
	Getting $\frac{1}{2}(A + A') = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$	1									
	Getting $\frac{1}{2}(A - A') = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = Q$ and Getting $P+Q = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$	1									
38	Writing, $\frac{dx}{d\theta} = a(1 - \cos\theta)$	1									
	Writing, $\frac{dy}{d\theta} = -a \sin\theta$	1									

	Getting $\frac{dy}{dx} = \frac{-a \sin \theta}{a(1 - \cos \theta)}$ OR Getting $\frac{dy}{dx} = -\cot \frac{\theta}{2}$	1
39	Writing, f is continuous in $[-2, 2]$ and f is differentiable in $(-2, 2)$	1
	Getting $f'(x) = 2x$ OR Getting $f(-2) = 6$ and $f(2) = 6$	1
	Writing $c = 0 \in (-2, 2)$	1
40	Writing, $f'(x) = 4x - 3$ OR Getting $x = \frac{3}{4}$	1
	Writing, f is increasing in $(\frac{3}{4}, \infty)$	1
	Writing, f is decreasing in $(-\infty, \frac{3}{4})$	1
41	Writing, $\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$	1
	Getting, $A = 1$ and $B = -1$	1
	Getting, $I = \log(x+1) - \log(x+2) + c$ OR $I = \log \left \frac{x+1}{x+2} \right + c$	1
42	Writing, $I = x \int \sin 3x dx - \int \left(\frac{d}{dx}(x) \cdot \int \sin 3x dx \right) dx$	1
	Getting, $I = -x \frac{\cos 3x}{3} + \int \frac{\cos 3x}{3} dx$	1
	Getting $I = -x \frac{\cos 3x}{3} + \frac{\sin 3x}{9} + c$	1
43	 OR Writing Area, $A = \int_2^4 x dy = \int_2^4 2\sqrt{y} dy$	1
	Writing, Area, $A = 2 \left[\frac{3}{2} y^{\frac{3}{2}} \right]_2^4$	1
	Getting, Area = $\frac{4}{3} \left[4^{\frac{3}{2}} - 2^{\frac{3}{2}} \right]$ OR Getting $\frac{4}{3} [8 - 2\sqrt{2}]$	1
44	Writing, $\frac{dy}{1+y^2} = (1+x^2)dx$	1
	Writing, $\int \frac{dy}{1+y^2} = \int (1+x^2)dx$	1
	Getting, $\tan^{-1} y = x + \frac{x^3}{3} + C$	1
45	 OR Writing $\frac{m}{n} = \frac{AP}{PB}$	1
	Writing $m(\overrightarrow{OB} - \overrightarrow{OP}) = n(\overrightarrow{OP} - \overrightarrow{OA})$	1
	Getting $\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$ OR $\overrightarrow{OP} = \frac{m\vec{b} + n\vec{a}}{m+n}$	1
46	Writing, LHS = $(\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})]$	1

	Writing, $(\vec{a} + \vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a}]$	1
	Getting, RHS = $\vec{a} \cdot (\vec{b} \times \vec{c}) + 0 + 0 + 0 + 0 + \vec{b} \cdot (\vec{c} \times \vec{a}) = 2[\vec{a} \vec{b} \vec{c}]$	1
47	Writing, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ OR $(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}$ OR $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$ OR $ (\vec{a}_2 - \vec{a}_1) \times \vec{b} = \sqrt{293}$ OR $ \vec{b} = 7$	1
	Writing, $d = \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} }$	1
	Getting, $d = \frac{\sqrt{293}}{7}$	1
48	$P(E_1) = \frac{60}{100} = \frac{3}{5}$, $P(E_2) = \frac{40}{100} = \frac{2}{5}$, $P(A E_1) = \frac{30}{100} = \frac{3}{10}$ $P(A E_2) = \frac{20}{100} = \frac{1}{5}$	1
	Writing $P(E_1 A) = \frac{P(A E_1)P(E_1)}{P(A E_1)P(E_1) + P(A E_2)P(E_2)}$	1
	Getting, $P(E_1 A) = \frac{9}{13}$	1
PART D		
49	Writing $f(x) = y = 4x + 3$ and getting $g(y) = \frac{y-3}{4}$	1
	Showing $f \circ g(y) = y$	1
	Showing $g \circ f(x) = x$	1
	Writing $f \circ g = I_Y$ and $g \circ f = I_X$ and concluding f is invertible or f^{-1} inverse exists.	1
	Getting $f^{-1}(x) = \frac{x-3}{4}$ or Writing g is the inverse of f	1
	OR	
	Proving $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$	1
	Hence f is one-one	1
	Writing $x = \frac{y-3}{4}$	1
	$\forall y \in \mathbb{R}$ there exist $x \in \mathbb{R}$ such that $f(x) = y$ hence f is onto OR Proving $f\left(\frac{y-3}{4}\right) = y$	1
	Writing f is invertible and getting $f^{-1}(x) = \frac{x-3}{4}$	1
50	Getting $A+B = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 7 & -8 & 0 \end{bmatrix}$	1
	Getting $AC = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$	1
	Getting $BC = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$	1

	Getting $(A+B)C = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$	1
	Getting $AC+BC = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$ and hence $(A+B)C=AC+BC$	1
51	Writing $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ OR Getting $ A = -17 \neq 0$ Note: Award a mark, if student writes directly $ A = -17$.	1
	Getting $\text{adj}(A) = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$ Note: If any 4 cofactors are correct award 1 mark.	2
	Writing $X = A^{-1}B = \frac{1}{ A }(\text{adj}A)B$ OR $X = -\frac{1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1
	Getting $x = 1$, $y = 2$, $z = 3$	1
52	Getting: $\frac{dy}{dx} = 6e^{2x} + 6e^{3x}$	1
	Getting: $\frac{d^2y}{dx^2} = 6e^{2x}(2) + 6e^{3x}(3)$	1
	Writing $\frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x}$	1
	Substituting the values of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$, and y in the LHS	1
	Getting $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$	1
53	Writing $\frac{dx}{dt} = -5$ and $\frac{dy}{dt} = 4$	1
	Writing Perimeter of the rectangle, $P = 2(x + y)$	1
	Getting $\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2(-5 + 4) = -2$	1
	Writing Area of the rectangle, $A = x \cdot y$	1
	Getting $\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} = 8(4) + 6(-5) = 2$	1
54	Taking: $x = a \tan \theta$	1
	Writing $I = \int \frac{a \sec^2 \theta d\theta}{\sqrt{a^2 \tan^2 \theta + a^2}} = \int \sec \theta d\theta$	1
	$I = \log x + \sqrt{x^2 + a^2} - \log a + c_1 = \log x + \sqrt{x^2 + a^2} + c$,	1
	Writing $\int \frac{1}{\sqrt{x^2+2x+2}} dx = \int \frac{1}{\sqrt{(x+1)^2+1}} dx$	1

	Getting $\log \left (x+1) + \sqrt{(x+1)^2 + 1} \right + C$	1
55	Correct Figure: 	1
	Writing Area $A = 4 \int_0^a y dx$	1
	Writing Area $A = 4 \int_0^a \sqrt{a^2 - x^2} dx$	1
	Getting Area $A = 4 \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a$	1
	Getting Area $= \pi a^2$ square unit. Note: Units are not compulsory	1
	56	Writing $\frac{dy}{dx} + \frac{2}{x}y = x$ OR Writing $P = \frac{2}{x}$ and $Q = x$
Getting $I.F. = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log(x)} = x^2$		1
Writing $\therefore y(I.F.) = \int Q(I.F.) dx + C$		1
Getting $yx^2 = \int x \cdot x^2 dx + C$ OR $yx^2 = \int x^3 dx + C$		1
Getting general solution $yx^2 = \frac{x^4}{4} + C.$		1
57		1
	Correct figure: Note: Award mark for other correct figure writing w.r.t. Co-ordinate axis.	
	Writing $\overrightarrow{AP} \cdot \vec{N} = 0$	1
	Writing $(\overrightarrow{OP} - \overrightarrow{OA}) \cdot \vec{N} = 0$ and $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$	1

	Writing $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$	1
	Getting: Cartesian form $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$	1
58	$n=6$, $p = \frac{1}{2}$ and $q = \frac{1}{2}$	1
	Writing $P(X = x) = nC_x p^x q^{n-x}$	1
	Getting $P(x=5) = 6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$	1
	$P(X = 5) + P(X = 6) = 6C_5 \left(\frac{1}{2}\right)^6 + 6C_6 \left(\frac{1}{2}\right)^6 = \frac{7}{64}$	1
	Getting $P(X \leq 5) = 1 - P(X = 6) = 1 - 6C_6 \left(\frac{1}{2}\right)^6 = 1 - \frac{1}{64} = \frac{63}{64}$	1

PART E

59		<p>Drawing the graph of 2 lines carries 2 mark and shading the feasible region carries 1 mark</p>	2+1=3										
	Getting corner points A(0,4), D(4,0), E(2,3) and O(0,0)		1										
	<table border="1" style="width: 100%;"> <thead> <tr> <th>Corner points</th> <th>$Z = -3x + 4y$</th> </tr> </thead> <tbody> <tr> <td>A(0,4)</td> <td>16</td> </tr> <tr> <td>D(4,0)</td> <td>-12</td> </tr> <tr> <td>E(2,3)</td> <td>6</td> </tr> <tr> <td>O(0,0)</td> <td>0</td> </tr> </tbody> </table>	Corner points	$Z = -3x + 4y$	A(0,4)	16	D(4,0)	-12	E(2,3)	6	O(0,0)	0		1
	Corner points	$Z = -3x + 4y$											
	A(0,4)	16											
D(4,0)	-12												
E(2,3)	6												
O(0,0)	0												
Minimum of Z is -12 at ((4,0) OR showing it in the above second row		1											

OR

	$I = \int_0^a f(x) dx$ <p>Putting $x = a - t$, then $dx = -dt$ and $x = 0$, then $t = a$ and $x = a$, then $t = 0$</p>	1
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	Getting $I = -\int_a^0 f(a-t) dt$	1
	Getting $I = \int_0^a f(a-x) dx$	1
	Writing $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$. and Getting $I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$	1
	Getting $2I = \int_0^{\frac{\pi}{2}} 1 dx$	1
	Getting $I = \frac{\pi}{4}$	1
60	Writing $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$	1
	Getting $\lim_{x \rightarrow 2^-} f(x) = 4k$	1
	Getting $\lim_{x \rightarrow 2^+} f(x) = 3$	1
	Getting $k = \frac{3}{4}$	1
OR		
	Applying $R_1 \rightarrow R_1 + R_2 + R_3$ and getting $\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$	1
	$a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$	1
	Applying $c_2 = c_2 - c_1$ and $c_3 \rightarrow c_3 - c_1$ and getting $a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$	1
	On expansion getting $(a+b+c)^3$	1
